An Approach to Financial Planning of Retirement Pensions with Scenario-Dependent Correlation Matrixes and Convex Risk Measures

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HOW SHOULD COMPANIES FUND THEIR LIABILITIES AND DETERMINE ALLOCATIONS AMONG ASSET CLASSES AND HEDGING INSTRUMENTS?

Siemens AG Österreich, part of the global Siemens Corporation, is the largest privately owned industrial company in Austria. Its businesses, with revenues of €2.4 billion in 1999, include information and communication networks, information and communication products, business services, energy and traveling technology, and medical equipment. Their pension fund, established in 1998, is a defined contribution plan and the largest corporate pension plan in Austria. More than 15,000 employees and 5,000 pensioners are members of the pension plan, which has €510 million in assets under management as of December 1999.

Innovest Finanzdienstleistungs AG, founded in 1998, is the investment manager for Siemens AG Österreich, the Siemens Pension Plan, and other institutional investors in Austria. With €2.2 billion in assets under management, Innovest focuses on asset management for institutional money and pension funds. This pension plan was rated the best in Austria of the 17 plans analyzed in the 1999–2000 period. The motivation to build InnoALM, which is described in Geyer and Ziemba [2008], stems from their desire for superior performance and good decision aids to help achieve this result.

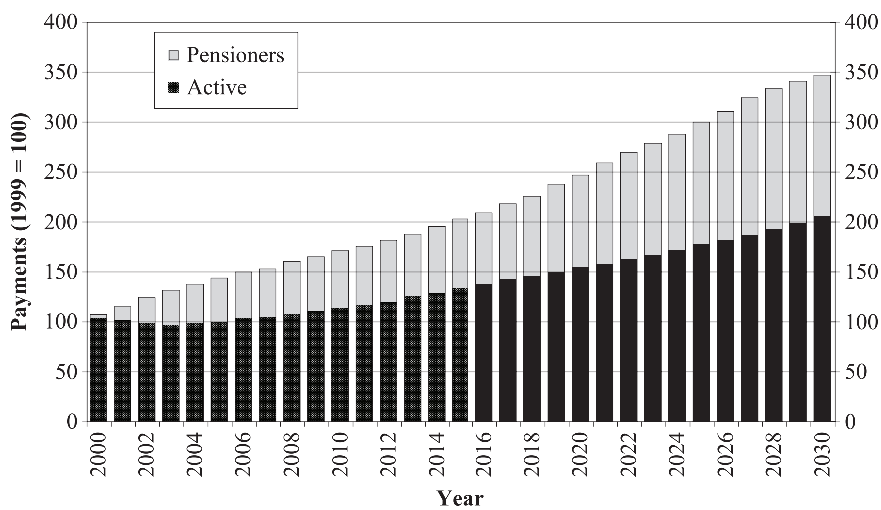
Various uncertain aspects—such as future economic scenarios, performance of stock, bond and other investments, transaction costs, liquidity, currency movements, liability commitments over time, Austrian pension fund law, and company policy—pointed to use of a multiperiod stochastic linear programming model. These models evolve from Kusy and Ziemba [1986] and Ziemba and Mulvey [1998], and the Russell–Yasuda Kasai model (see Cariño et al. [1994] and Cariño et al. [1998]). Other stochastic programming applications are discussed in Consiglio et al. [2004]; Wallace and Ziemba [2005]; Consiglio et al. [2006]; Zenios and Ziemba [2006, 2007]; Zenios [2007]; Bertocchi et al. [2010]; Consigli et al. [2012]; and Consiglio et al. [2015]. Cariño and Turner [1998] discuss having derivative securities in the Russell–Yasuda Kasai-type model. This model has innovative features such as state-dependent correlation matrixes, fat-tailed asset return distributions, a convex risk measure leading to a concave maximization problem, and simple computational schemes and output. The case for convex risk measures over Var and C-Var in asset-liability models is made in Ziemba [2013]. Ziemba [2003] discusses the application of these models to other types of financial institutions with similar models, such as the InnoALM detailed here. Birge and Louveaux [2011] discuss stochastic programming theory.

InnoALM was produced in six months during 2000 with Geyer and Ziemba serving as consultants with Herold and Kontriner, the Innovest employees. InnoALM demonstrates that a small team of researchers with a limited budget can quickly produce a valuable modeling system that can easily be operated by non-stochastic programming specialists on a single PC. The IBM OSL stochastic programming software provides a good solver. The solver was interfaced with user-friendly input and output capabilities. Calculation times on the PC are such that different modeling situations can be easily developed and the implications of policy, scenario, and other changes can be seen quickly. The graphical output provides pension fund management with essential information to aid in the making of informed investment decisions and to understand the probable outcomes and risk involved with these actions. The model can be used to explore possible European, Austrian, and Innovest policy alternatives.

The liability side of the Siemens Pension Plan consists of employees, for whom Siemens is contributing defined contribution payments, and retired employees who receive pension payments. Contributions are based on a fixed fraction of salaries, which varies across employees. Active employees are assumed to be in steady state; thus, employees are replaced by a new employee with the same qualification, sex, and age group characteristics so there is a constant number of similar employees. Newly employed staff start with less salary than retired staff, which implies that total contributions grow less rapidly than individual salaries. Exhibit 1 shows the expected index of total payments for active and retired employees until 2030.

The set of retired employees is modeled using Austrian mortality and marital tables. Widows receive 60% of the pension payments. Retired employees receive pension payments after reaching age 65 for men and 60 for women. Payments to retired employees are based upon the individually accumulated contribution and the fund performance during active employment. The annual pension payments are based on a discount rate of 6% and the remaining life expectancy at the time of retirement. These annuities grow by 1.5% annually to compensate for inflation. Hence, the wealth of the pension fund must grow by 6% for the growth in employees and 1.5% for inflation, or 7.5% per year in total, to match liability commitments. Another output of the computations is the expected annual net cash flow of plan contributions minus payments. Since the number of pensioners is rising faster than plan contributions, these cash flows are negative, so the plan is declining in size.

EXHIBIT 1
Index of Expected Payments for Active and Retired Employees, 2000–2030



Source: Geyer and Ziemba [2008].

The model determines the optimal non-negative purchases (P) and sales (S) for each of N assets in each of T planning periods. Typical asset classes used at Innovest are U.S., Pacific, European, and emerging market equities and U.S., U.K., Japanese, and European bonds. The objective is to maximize the concave risk averse utility function: expected terminal wealth less convex penalty costs subject to various linear constraints. The effect of such constraints is evaluated in the examples that follow, including Austria's limits of 40% maximum in equities, 45% maximum in foreign securities, and 40% minimum in Eurobonds. The convex risk measure is approximated by a piecewise linear function, so the model is a multiperiod stochastic linear program. Typical targets that the model tries to achieve (and if not, is penalized for), are for wealth (the fund's assets) to grow by 7.5% per year and for portfolio performance returns to exceed benchmarks. The former is a deterministic target, while the latter is a stochastic target. The 7.5% target accounts for inflation in pension payments plus the growth of the number of pensioners actually receiving benefits. Excess wealth is placed into surplus reserves, and a portion of it is paid out in succeeding years.

The penalty costs serve to force the allocations to comply with these targets. In the Russell–Yasuda Kasai model, David Myers (see Cariño et al. [1998]) calculated the exact cost of penalty violations, adding up all the costs; we used these results for the coefficients. Since the InnoALM model was intended for many applications, we allow the user to set the costs at realistic values, which are likely high enough to force non-violations of these targets.

The elements of InnoALM are described in Exhibit 2. The interface to read in data and problem elements uses Excel. Statistical calculations use the program Gauss, and this data is fed into the IBM0SL solver, which generates the optimal solution to the stochastic program. The output, some of which is shown in the next section, used Gauss to generate various tables and graphs and retains key variables in memory to allow for future modeling calculations.

FORMULATING INNOALM AS A MULTISTAGE STOCHASTIC LINEAR PROGRAMMING MODEL

The non-negative decision variables are wealth (after transactions costs), and purchases and sales for each asset ($i = 1, \ldots, N$). Purchases and sales take place in periods $t = 0, \ldots, T–1$. Except for $t = 0$, purchases and sales are scenario dependent. All decision variables are non-negative.

Wealth accumulates over time for a T period model according to

$$\begin{aligned}
W_{i0} &= W_i^{init} + P_{i0} - S_{i0}, && t = 0\\
\tilde{W}_{it} &= \tilde{R}W_{i0} + \tilde{P}_{i1} - \tilde{S}_{i1}, && t = 1\\
\tilde{W}_{it} &= \tilde{R}_{it}W_{i,t-1} + \tilde{P}_{it} - \tilde{S}_{it}, && t = 2, \ldots, T-1,\\
\tilde{W}_{iT} &= \tilde{R}_{i,T-1}W_{ii,T-1}, && t = T,
\end{aligned}$$

where W_i^{init} is the initial value of asset i. There is no uncertainty in the initialization period $t = 0$. Tildes

EXHIBIT 2

Elements of InnoALM: GAUSS is a Statistical Package, IBMOSL Solver is the Code Used to Compute the Stochastic Programming Solutions, and SMPS Presents the Model in Readable Form into the Optimization Code

Front-end user interface (Excel)
Periods (targets, node structure, fixed cash-flows, ...)
Assets (selection, distribution, initial values, transaction costs, ...)
Liability data
Statistics (mean, standard deviation, correlation)
Bounds
Weights
Historical data
Options (plot, print, save, ...)
Controls (breakpoints of cost function, random seed, ...)

⇓

GAUSS
Read input
Compute statistics
Simulate returns and generate scenarios
Generate SMPS files (*core*, *stoch* and *time*)

⇓

IBMOSL solver
Read SMPS input files
Solve the problem
Generate output file (optimal solutions for all nodes and variables)

⇓

Output interface (GAUSS)
Read optimal solutions
Generate tables and graphs
Retain key variables in memory to allow for further analyses

Source: Geyer and Ziemba [2008].

denote random scenario-dependent parameters or decision variables. Returns $\tilde{R}_{it}$ ($t = 1, \ldots, T$) are the gross returns from asset i between $t = 1$ and t.

The budget constraints are

$$\sum_{i=1}^{N} P_{i0}(1 + tcp_i) = \sum_{i=1}^{N} S_{i0}(1 - tcs_i) + C_0 \qquad t = 0, \textit{ and}$$

$$\sum_{i=1}^{N} \tilde{P}_{it}(1 + tcp_i) = \sum_{i=1}^{N} \tilde{S}_{it}(1 - tcs_i) + C_i \qquad t = 1, \ldots, T - 1,$$

where tcp_i and tcs_i are the linear transaction costs for purchases and sales, and C_i is the fixed (non-random) net cashflow (inflow, if positive).

Since short sales are not allowed, the following constraints are included

$$S_{i0} \leq W_i^{init} \qquad i = 1, \ldots, N; t = 0, \textit{ and}$$

$$\tilde{S}_{it} \leq \tilde{R}_i \tilde{W}_{i,t-1} \qquad i = 1, \ldots, N; t = 1, \ldots, T - 1.$$

Portfolio weights can be constrained over linear combinations (subsets) of assets or individual assets via

$$\sum_{i \in U_t} \tilde{W}_{it} - \theta_U \sum_{i=1}^{N} \tilde{W}_{it} \geq 0, \textit{ and}$$

$$\sum_{i \in L_t} \tilde{W}_{it} - \theta_L \sum_{i=1}^{N} \tilde{W}_{it} \geq 0, \qquad t = 0, \ldots, T - 1$$

where θ_U is the maximum percentage and θ_L is the minimum percentage of the subsets U_j and L_l of assets $i = 1, \ldots, N$ included in the restrictions j and l, respectively. The θ_U's, θ_L's, U_j's and L_l's may be time dependent. Austria, Germany, and other European Union countries have restrictions that vary from country to country but not across time. Austria currently has the following limits: max 40% equities, max 45% foreign securities, min 40% Eurobonds, max 5% total premiums in non-currency hedge options short and long positions. The model has convex penalty risk function costs if goals in each period are not satisfied. In a typical application, the wealth target $\bar{W}_t$ is assumed to grow by 7.5% in each period. This is a deterministic target goal for the increase in the pension funds assets. The wealth targets are modeled via

$$\sum_{i=1}^{N} (\tilde{W}_{it} - \tilde{P}_{it} + \tilde{S}_{it}) + \tilde{M}_t^W \geq \bar{W}_t \qquad t = 1, \ldots, T,$$

where $\tilde{M}_t^W$ are wealth-target shortfall variables. The shortfall (or embarrassment) is penalized using a piecewise linear risk measure based on the variables and constraints

$$\tilde{M}_t^W = \sum_{j=1}^{m} \tilde{M}_{jt}^W, \qquad t = 1, \ldots, T$$

$$\tilde{M}_{jt}^W \leq b_j - b_{j-1}, \qquad t = 1, \ldots, T; \; j = 1, \ldots, m - 1,$$

where $\tilde{M}_{jt}^W$ is the wealth target shortfall associated with segment j of the cost-function, b_j is the j-th breakpoint of the risk measure function $b_0 = 0$, and m is the number of segments of the function. A quadratic function works well but other functions may be linearized as well. Convexity guarantees that if $\tilde{M}_{jt}^W \geq 0$ then $\tilde{M}_{j-1,t}^W$ is at its maximum and if $\tilde{M}_{jt}^W$ is not at its maximum then $\tilde{M}_{j+1,t}^W = 0$.

Stochastic benchmark goals can be set by the user and are similarly penalized with a piecewise linear convex risk measure for underachievement. The benchmark target $\tilde{B}_t$ is scenario dependent. It is based on stochastic asset returns and fixed asset weights defining the benchmark portfolio

$$\tilde{B}_t = W_0 \sum_{j=1}^{t} \sum_{i=1}^{N} \alpha_i \tilde{R}_{it}$$

with shortfall constraints

$$\sum_{i=1}^{N} \tilde{W}_{it} + \tilde{M}_t^B \geq \tilde{B}_t \qquad t = 1, \ldots, T,$$

where $\tilde{M}_t^B$ is the benchmark-target shortfall.

If the total wealth implied by the allocation is above the target a percentage γ, typically a conservative 10% of the exceeding amount is allocated to the reserve account. Then the wealth targets for all future stages are increased. For this purpose, additional non-negative decision variables $\tilde{D}_t$ are introduced, and the wealth target constraints become

$$\sum_{i=1}^{N} (\tilde{W}_{it} - \tilde{P}_{it} + \tilde{S}_{it}) - \tilde{D}_t + \tilde{M}_t^W$$
$$= \bar{W}_t + \sum_{j=1}^{t-1} \gamma \tilde{D}_{t-j}, \qquad t = 1, \ldots, T - 1, \text{ where } \tilde{D}_1 = 0.$$

Since pension payments are based on wealth levels, increasing these levels increases pension payments. The reserves provide security for the pension plan's increase of pension payments at each future stage. The fund had accumulated such a surplus by 2000.

The pension plan's objective function is to maximize the expected discounted value of terminal wealth in period T net of the expected discounted penalty costs over the horizon from the convex risk measures $c_k(\cdot)$ for the wealth and benchmark targets, respectively,

$$\text{Max } E\left[d_T \sum_{i=1}^{N} \tilde{W}_{iT} - \lambda \sum_{t=1}^{T} d_t w_t \left(\sum_{k \in \{W,B\}} v_k c_k(\tilde{M}_t^k) \right) \right].$$

Expectation is over T period scenarios S_T. The v_k are weights for the wealth and benchmark shortfalls, and the w_t are weights for the weighted sum of shortfalls at each stage. The weights are normalized via

$$\sum_{k \in \{W,B\}} v_k = 1 \quad \text{and} \quad \sum_{t=1}^{T} w_t = T.$$

The discount factors d_t are defined on the basis of an interest rate r: $d_t = (1 + r)^{-t}$. Usually r is taken to be the three- or six-month Treasury-bill rate. However, Campbell and Viceira [2002] argue that, in a multiperiod world, the proper risk-free asset is an inflation-indexed annuity, rather than the short-dated T-bill. Their analysis is based on a model in which agents desire to hedge against unanticipated changes in the real rate of interest. Ten-year inflation-index bonds are then suggested for r as their duration closely approximates the indexed annuity.

The shortfall cost coefficients are based on the least cost way to make up the shortfall-embarrassments, which may be the product of an optimized combination of borrowing, equity, short- and long-term debt, and other financial instruments. See Cariño et al. [1994]; Cariño et al. [1998]; and Cariño and Ziemba [1998] for a discussion.

Allocations are based on optimizing the stochastic linear program with IBM's optimization solutions library using the stochastic extension library (OSLE version 3).[1] The library uses the Stochastic Mathematical Programming System (SMPS) input format for multistage stochastic programs (see Birge et al. [1986]). The OSLE routines require three input files: the core-, stoch- and time-file. The core-file contains information about the decisions variables, constraints, right-hand-sides, and bounds. It contains all fixed coefficients and dummy entries for random elements. The stoch-file reflects the node structure of the scenario tree and contains all random elements, i.e., asset and benchmark returns, and probabilities. Non-anticipatory constraints are imposed to guarantee that a decision made at a specific node is identical for all scenarios leaving that node, so the future cannot be anticipated. This is implemented by specifying an appropriate scenario structure in the stoch input file. The time-file assigns decision variables and constraints to stages. The required statements in the input files are automatically generated by the InnoALM system.

SOME TYPICAL APPLICATIONS

To illustrate the model's use, I present results for a problem with four asset classes (Stocks Europe, Stocks US, Bonds Europe, and Bonds US) with five periods (six stages). The periods are twice 1 year, twice 2 years and 4 years (10 years in total). Discrete compounding is assumed, which implies that the mean return for asset i (μ_i) used in simulations is $\mu_i = exp(\overline{y})_i - 1$, where $\overline{y}_i$ is the mean based on log-returns. Next, 10,000 scenarios are generated using a 100-5-5-2-2 node structure. Initial wealth equals 100 units, and the wealth target is assumed to grow at an annual rate of 7.5%. No benchmark target and no cash in- and outflows are considered in this sample application to make its results more general. $R_A = 4$ and the discount factor equals 5%, which corresponds roughly with a simple static mean-variance model to a standard 60–40 stock-bond pension fund mix (see Kallberg and Ziemba [1983]).

Assumptions about the statistical properties of returns measured in nominal euros are based on a sample of monthly data from January 1970 for stocks and 1986 for bonds to September 2000. Summary statistics for monthly and annual log returns are in Exhibit 3. The U.S. and European equity means for the longer period 1970–2000 are much lower than for the period 1986–2000 and slightly less volatile. The monthly stock returns are non-normal and negatively skewed. Monthly stock returns are fat-tailed, whereas monthly bond returns are close to normal (the critical value of the Jarque-Bera test for a = .01 is 9.2).

However, for long-term planning models, such as InnoALM with its one year review period, properties of monthly returns are less relevant. The bottom panel of Exhibit 3 contains statistics for annual returns. While average returns and volatilities remain about the same (we lose one year of data, when we compute annual

Exhibit 3

Statistical Properties of Asset Returns

	Stocks Europe		Stocks U.S.		Bonds Europe	Bonds U.S.
	1/70 –9/00	1/86 –9/00	1/70 –9/0	1/86 –9/00	1/86 –9/00	1/86 –9/00
Monthly Returns						
Mean (% p.a.)	10.6	13.3	10.7	14.8	6.5	7.2
Std. Dev (% p.a.)	16.1	17.4	19.0	20.2	3.7	11.3
Skewness	–0.90	–1.43	–0.72	–1.04	–0.50	0.52
Kurtosis	7.05	8.43	5.79	7.09	3.25	3.30
Jarque–Bera test	302.6	277.3	151.9	155.6	7.7	8.5
Annual Returns						
Mean (%)	11.1	13.3	11.0	15.2	6.5	6.9
Std. Dev (%)	17.2	16.2	20.1	18.4	4.8	12.1
Skewness	–0.53	–0.10	–0.23	–0.28	–0.20	–0.42
Kurtosis	3.23	2.28	2.56	2.45	2.25	2.26
Jarque–Bera test	17.4	3.9	6.2	4.2	5.0	8.7

Source: Geyer and Ziemba [2008].

returns), the distributional properties change dramatically. Although we still find negative skewness, there is no evidence for fat tails in annual returns except for European stocks (1970–2000) and U.S. bonds.

The mean returns from this sample are comparable to the 1900–2000 one hundred and one year mean returns estimated by Dimson et al. [2002]. Their estimate of the nominal mean equity return for the United States is 12.0% and that for Germany and the United Kingdom is about 13.6% (the simple average of the means of the two countries). The mean of bond returns is 5.1% for the United States and 5.4% for Germany and the United Kingdom.

Assumptions about means, standard deviations, and correlations for the applications of InnoALM appear in Exhibit 5 and are based on the sample statistics in Exhibit 4. Projecting future rates of returns from past data is difficult. The equity means from the period 1970–2000 are used since the period 1986–2000 had exceptionally good performance of stocks that is not assumed to prevail in the long run.

The correlation matrixes in Exhibit 5 for the three different regimes are based on the regression approach of Solnik et al. [1996]. Moving average estimates of correlations among all assets are functions of standard deviations of U.S. equity returns. The estimated regression equations are then used to predict the correlations

Exhibit 4

Regression Equations Relating Asset Correlations and U.S. Stock Return Volatility (monthly returns; Jan 1989–Sep 2000; 141 observations)

Correlation Between	Constant	Slope w.r.t. U.S. stock Volatility	*t*-statistic of Slope	*R*
Stocks Europe–Stocks U.S.	0.62	2.7	6.5	0.23
Stocks Europe–Bonds Europe	1.05	–14.4	–16.9	0.67
Stocks Europe–Bonds U.S.	0.86	–7.0	–9.7	0.40
Stocks US–Bonds Europe	1.11	–16.5	–25.2	0.82
Stocks US–Bonds U.S.	1.07	–5.7	–11.2	0.48
Bonds Europe–Bonds U.S.	1.10	–15.4	–12.8	0.54

Source: Geyer and Ziemba [2008].

Exhibit 5

Means, Standard Deviations, and Correlations Assumptions

		Stocks Europe	Stocks U.S.	Bonds Europe	Bonds U.S.
Normal Periods	Stocks U.S.	.755			
(70% of the time)	Bonds Europe	.334	.286		
	Bonds U.S.	.514	.780	.333	
	Standard Deviation	14.6	17.3	3.3	10.9
high Volatility	Stocks U.S.	.786			
(20% of the time)	Bonds Europe	.171	.100		
	Bonds U.S.	.435	.715	.159	
	Standard Deviation	19.2	21.1	4.1	12.4
Extreme Periods	Stocks U.S.	.832			
(10% of the time)	Bonds Europe	–.075	–.182		
	Bonds U.S.	.315	.618	–.104	
	Standard Deviation	21.7	27.1	4.4	12.9
Average Period	Stocks U.S.	.769			
	Bonds Europe	.261	.202		
	Bonds U.S.	.478	.751	.255	
	Standard Deviation	16.4	19.3	3.6	11.4
All Periods	Mean	10.6	10.7	6.5	7.2

Source: Geyer and Ziemba [2008].

in the three regimes shown in Exhibit 5. Results for the estimated regression equations appear in Exhibit 4. Three regimes are considered, with the assumption that 10% of the time, equity markets are extremely volatile; 20% of the time, markets are characterized by high volatility; and

70% of the time, markets are normal. The 35% quantile of U.S. equity return volatility defines *normal* periods. *Highly volatile* periods are based on the 80% volatility quantile and *extreme* periods on the 95% quartile. The associated correlations reflect the return relationships that typically prevailed during those market conditions. The correlations in Exhibit 5 show a distinct pattern across the three regimes. Correlations among stocks tend to increase as stock return volatility rises, whereas the correlations between stocks and bonds tend to decrease. European bonds may serve as a hedge for equities during extremely volatile periods since bonds and stocks returns, which are usually positively correlated, are then negatively correlated. The latter is a major reason why using scenario-dependent correlation matrixes is a major advance over sensitivity of one correlation matrix.

Optimal portfolios were calculated for seven cases—with and without mixing of correlations and with normal, *t*-, and historical distributions. Cases NM, HM, and TM use mixing correlations. Case NM assumes normal distributions for all assets. Case HM uses the historical distributions of each asset. Case TM assumes *t*-distributions with five degrees of freedom for stock returns, and normal distributions for bond returns. The cases NA, HA, and TA use the same distribution assumptions with no mixing of correlations matrixes. Instead the correlations and standard deviations used in these cases correspond to an "average" period where 10%, 20%, and 70% weights are used to compute averages of correlations and standard deviations used in the three different regimes. Comparisons of the average (A) cases and mixing (M) cases are mainly intended to investigate the effect of mixing correlations. TMC maintains all assumptions of case TM but uses Austria's constraints on asset weights. Eurobonds must be at least 40%, and equity at most 40%, and these constraints are binding.

SOME TEST RESULTS

Exhibit 6 shows the optimal initial asset weights at stage 1 for the various cases. Exhibit 7 shows results for the final stage (expected weights, expected terminal wealth, expected reserves, and shortfall probabilities). These exhibits show a distinct pattern: the mixing correlation cases initially assign a much lower weight to European bonds than the average period cases. Single-period, mean-variance optimization, and the average period cases (NA, HA, and TA) suggest an approximate 45–55 mix between equities and bonds. The mixing correlation cases (NM, HM, and TM) imply a 65–35 mix. Investing in U.S. bonds is not optimal at stage 1 in any of the cases, which seems attributable to the relatively high volatility of U.S. bonds.

EXHIBIT 6
Optimal Initial Asset Weights at Stage 1 by Case (percentage)

	Stocks Europe	Stocks U.S.	Bonds Europe	Bonds U.S.
Single-period, mean-variance optimal weights (average periods)	34.8	9.6	55.6	0.0
Case NA: no mixing (average periods) normal distributions	27.2	10.5	62.3	0.0
Case HA: no mixing (average periods) historical distributions	40.0	4.1	55.9	0.0
Case TA: no mixing (average periods) *t*-distributions for stocks	44.2	1.1	54.7	0.0
Case NM: mixing correlations normal distributions	47.0	27.6	25.4	0.0
Case HM: mixing correlations historical distributions	37.9	25.2	36.8	0.0
Case TM: mixing correlations *t*-distributions for stocks	53.4	11.1	35.5	0.0
Case TMC: mixing correlations historical distributions; constraints on asset weights	35.1	4.9	60.0	0.0

Source: Geyer and Ziemba [2008].

EXHIBIT 7

Expected Portfolio Weights at the Final Stage by Case (percentage), Expected Terminal Wealth, Expected Reserves, and the Probability for Wealth Target Shortfalls (percentage) at the Final Stage

	Stocks Europe	Stocks U.S.	Bonds Europe	Bonds U.S.	Expected Terminal Wealth	Expected Reserves at Stage 6	Probability of Target Shortfall
NA	34.3	49.6	11.7	4.4	328.9	202.8	11.2
HA	33.5	48.1	13.6	4.8	328.9	205.2	13.7
TA	35.5	50.2	11.4	2.9	327.9	202.2	10.9
NM	38.0	49.7	8.3	4.0	349.8	240.1	9.3
HM	39.3	46.9	10.1	3.7	349.1	235.2	10.0
TM	38.1	51.5	7.4	2.9	342.8	226.6	8.3
TMC	20.4	20.8	46.3	12.4	253.1	86.9	16.1

Source: Geyer and Ziemba [2008].

Exhibit 7 shows that the distinction between A and M cases becomes less pronounced over time. However, European equities still have a consistently higher weight in the mixing cases than in no-mixing cases. This higher weight is mainly at the expense of Eurobonds. In general, the proportion of equities at the final stage is much higher than in the first stage. This result may be explained by the fact that the expected portfolio wealth at later stages is far above the target wealth level (206.1 at stage 6), and the higher risk associated with stocks is less important. The constraints in case TMC lead to lower expected portfolio wealth throughout the horizon and to a higher shortfall probability than in any other case. Calculations show that initial wealth would have to be 35% higher to compensate for the loss in terminal expected wealth owing to those constraints. In all cases, the optimal weight of equities is much higher than the historical 4.1% in Austria.

The expected terminal wealth levels and the shortfall probabilities at the final stage shown in Exhibit 7 make the difference between mixing and no-mixing cases even clearer. Mixing correlations yields higher levels of terminal wealth and lower shortfall probabilities.

If the level of portfolio wealth exceeds the target, the surplus $\tilde{D}$ is allocated to a reserve account. The reserves in t are computed from $\sum_{j=1}^{t} \widetilde{D}_j$ and as shown in Exhibit 7 for the final stage. These values are in monetary units, given an initial wealth level of 100. They can be compared with the wealth target 206.1 at stage 6. Expected reserves exceed the target level at the final stage by up to 16%. Depending on the scenario, the reserves can be as high as 1,800. Their standard deviation (across scenarios) ranges from 5 at the first stage to 200 at the final stage. The constraints in case TMC lead to a much lower level of reserves compared with the other cases, which implies, in fact, less security against future increases of pension payments.

Summarizing the optimal allocations, expected wealth, and shortfall probabilities are mainly affected by considering mixing correlations, whereas the type of distribution chosen has a smaller impact. This distinction is mainly attributable to the higher proportion allocated to equities if different market conditions are taken into account by mixing correlations.

The results of any asset allocation strategy crucially depend upon the mean returns. This effect is now investigated by parametrizing the forecasted future means of equity returns. Assume that an econometric model forecasts that the future mean return for U.S. equities is some value between 5% and 15%. The mean of European equities is adjusted accordingly so that the ratio of equity means and the mean bond returns as in Exhibit 5 are maintained. All other assumptions of case NM hold (normal distribution and mixing correlations). Exhibit 8 summarizes the effects of these mean changes in terms of the optimal initial weights. As expected (see Chopra and Ziemba [1993] and Kallberg and Ziemba [1981, 1983]), the results are very sensitive to the choice of the mean return. If we assume that the mean return for U.S. stocks is equal the long run mean of 12% as estimated by Dimson et al. [2002], the model yields an optimal weight for equities of 100%. However, a mean return for U.S. stocks of 9% implies less than 30% optimal weight for equities.

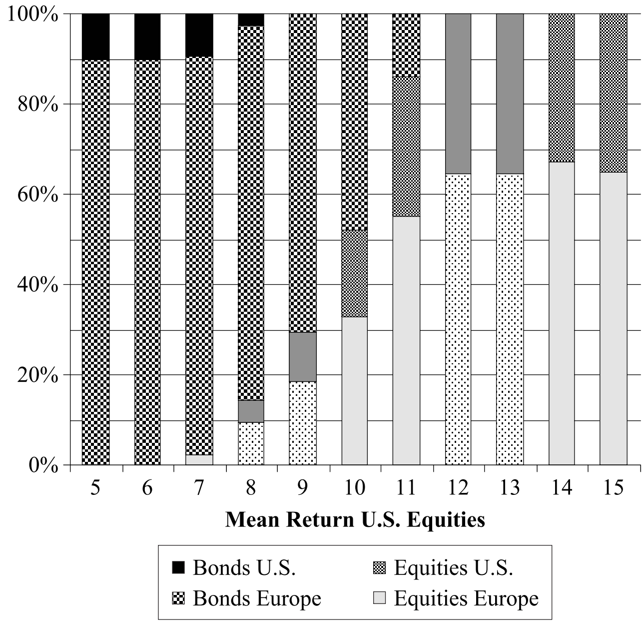
MODEL TESTS

Since state-dependent correlations have a significant impact on allocation decisions, it is worthwhile to further investigate their nature and implications from the perspective of testing the model. Positive effects on the pension fund performance induced by the stochastic, multiperiod planning approach will only be realized if the portfolio is dynamically rebalanced as implied by the optimal scenario tree. The performance of the model is tested considering this aspect. As a starting point, it is instructive to break down the rebalancing decisions at later stages into groups of achieved wealth levels. This approach reveals the "decision rule" implied by the model depending on the current state. Consider case TM: Quintiles of wealth are formed at stage 2 and the average optimal weights assigned to each quintile are computed. The same is done using quintiles of wealth at stage 5.

Exhibit 9 shows the distribution of weights for each of the five average levels of wealth at the two stages. While the average allocation at stage 5 is essentially independent of the wealth level achieved (the target wealth at stage 5 is 154.3), the distribution at stage 2 depends on the wealth level in a specific way. If average attained wealth is 103.4, which is slightly below the target, a very cautious strategy is chosen. Bonds have the highest weight in this case (almost 50%). In this situation, the model implies that the risk of even stronger underachievement of the target is to be minimized. The model relies on the low, but more certain, expected returns of bonds to move back to the target level. If attained wealth is far below the target (97.1), the model implies more than 70% equities and a high share (10.9%) of relatively risky U.S. bonds. With such strong underachievement, there is no room for a cautious strategy to attain the target level again. If average attained wealth

EXHIBIT 8

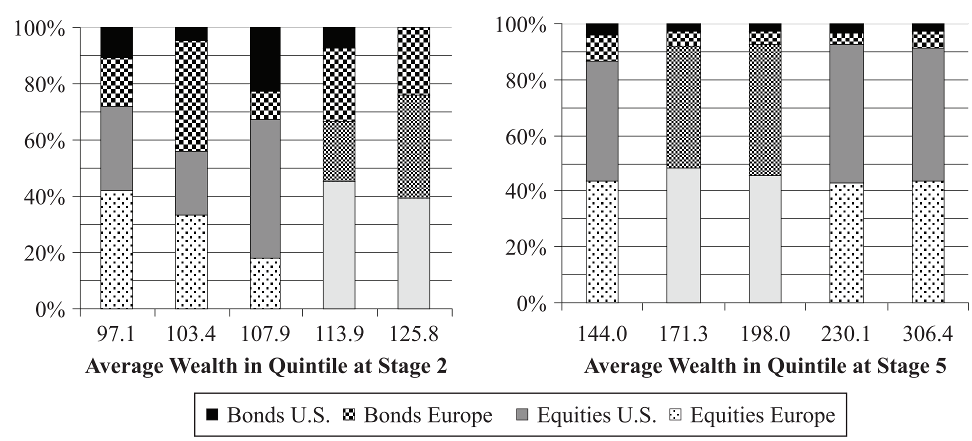
Optimal Asset Weights at Stage 1 for Varying Levels of U.S. Equity Means



Source: Geyer and Ziemba [2008].

EXHIBIT 9

Optimal Weights Conditional on Quintiles of Portfolio Wealth at Stage 2 and Stage 5



Source: Geyer and Ziemba [2008].

equals 107.9, which is close to the target wealth of 107.5, the highest proportion is invested into U.S. assets, with 49.6% invested in equities and 22.8% in bonds. The U.S. assets are more risky than the corresponding European assets, which is acceptable because portfolio wealth is very close to the target and risk does not play a big role. For wealth levels above the target, most of the portfolio is switched to European assets, which are safer than U.S. assets. This "decision" may be interpreted as an attempt to preserve the high levels of attained wealth.

The decision rules implied by the optimal solution can be used to perform a test of the model using the following rebalancing strategy. Consider the ten-year period from January 1992 to January 2002. In the first month of this period, we assume that wealth is allocated according to the optimal solution for stage 1 given in Exhibit 6. In each of the subsequent months, the portfolio is rebalanced as follows: identify the current volatility regime (extreme, highly volatile, or normal) based on the observed U.S. stock return volatility. Then search the scenario tree to find a node that corresponds to the current volatility regime and has the same or a similar level of wealth. The optimal weights from that node determine the rebalancing decision.

For the no-mixing cases NA, TA, and HA, the information about the current volatility regime cannot be used to identify optimal weights. In those cases, use the weights from a node with a level of wealth as close as possible to the current level of wealth. Exhibit 10 presents summary statistics for the complete sample and the out-of-sample period October 2000 to January 2002.

The mixing correlation solutions assuming normal and t-distributions (cases NM and TM) provide a higher average return with lower standard deviation than the corresponding non-mixing cases (NA and TA). The advantage may be substantial, as indicated by the 14.9% average return of TM compared to 10.0% for TA. The t-statistic for this difference is 1.7 and is significant at the 5% level (one-sided test). Using the historical distribution and mixing correlations (HM) yields a lower average return than no-mixing (HA). In the constrained case TMC, the average return for the complete sample is in the same range as that for the unconstrained cases. This is mainly owing to relatively high weights assigned to U.S. bonds which performed very well during the test period, whereas stocks performed poorly. The standard deviation of returns is much lower because the constraints imply a lower degree of rebalancing.

EXHIBIT 10
Results of Asset Allocation Strategies Using the *Decision Rule* Implied by the Optimal Scenario Tree

	Complete Sample 01/92–01/02		Out-of-Sample 10/00–01/02	
	Mean	Std. Dev.	Mean	Std. Dev.
NA	11.6	16.1	–17.1	18.6
NM	13.1	15.5	–9.6	16.9
HA	12.6	16.5	–15.7	21.1
HM	11.8	16.5	–15.8	19.3
TA	10.0	16.0	–14.6	18.9
TM	14.9	15.9	–10.8	17.6
TMC	12.4	8.5	0.6	9.9

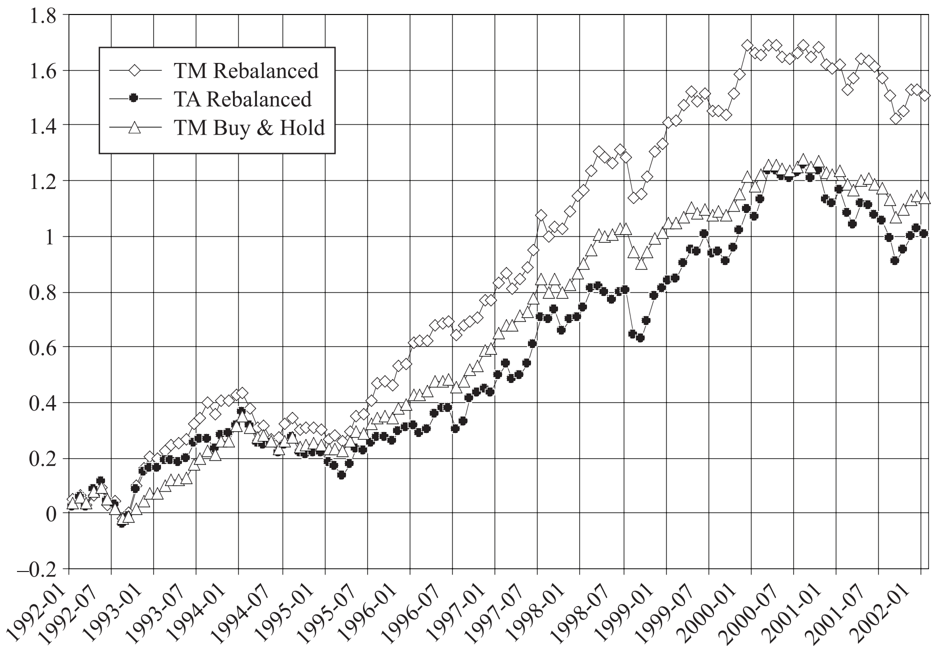
Source: Geyer and Ziemba [2008].

To emphasize the difference between the cases TM and TA, Exhibit 11 compares the cumulated monthly returns obtained from the rebalancing strategy for the two cases as well as a buy-and-hold strategy, which assumes that the portfolio weights on January 1992 are fixed at the optimal TM weights throughout the test period. Rebalancing on the basis of the optimal TM scenario tree provides a substantial gain when compared to the buy-and-hold strategy or the performance using TA results, in which rebalancing does not account for different correlation and volatility regimes.

Such in- and out-of-sample comparisons depend on the asset returns and test period. To isolate the potential benefits from considering state-dependent correlations, the following controlled simulation experiment was performed. Consider 1,000 ten-year periods in which simulated annual returns of the four assets are assumed to have the statistical properties summarized in Exhibit 5. One of the ten years is assumed to be an "extreme" year, two years correspond to "highly volatile" markets, and seven years are "normal" years. We compare the average annual return of two strategies: (a) a buy-and-hold strategy using the optimal TM weights from Exhibit 6 throughout the ten-year period, and (b) a rebalancing strategy that uses the implied decision rules of the optimal scenario tree as explained in the in- and out-of-sample tests above. For simplicity, it was assumed that the current volatility regime is known in each period. The average annual returns over 1,000 repetitions of the two strategies are 9.8% (rebalancing) and 9.2% (buy and hold). The t-statistic for the mean difference is 5.4 and

Exhibit 11

Cumulative Monthly Returns for Different Strategies



Source: Geyer and Ziemba [2008].

indicates a highly significant advantage of the rebalancing strategy, which exploits the information about state-dependent correlations. For comparison, the same experiment was repeated using the optimal weights from the constrained case TMC. The same average mean of 8.1% for both strategies was obtained. This result indicates that the constraints imply insufficient rebalancing capacity. Therefore, knowledge about the volatility regime cannot be sufficiently exploited to achieve superior performance relative to the buy-and-hold strategy. This result also shows that the relatively good performance of the TMC rebalancing strategy in the sample period 1992–2002 is positively biased by the favorable conditions during that time.

Final Comments

The model InnoALM provides a relatively easy-to-use tool to help Austrian pension funds' investment allocation committees evaluate the effects of various policy choices in light of changing economic conditions and various goals, constraints, and liability commitments. As suggested by a referee, it is flexible, sophisticated, and—given the state of the art in software availability—a computationally tractable computational tool. The clients have been able to use it in regulatory and pension fund applications as the quote below indicates. The model includes features that reflect real investment practices. These features include multiple scenarios, non-normal distributions, and different volatility and correlation regimes. The model provides a systematic way to estimate in advance the likely results of particular policy changes and asset return realizations. This model provides more confidence and justification to policy changes that may be controversial, such as a higher weight in equity and less in bonds than has traditionally been the case in Austria.

The model is an advance on previous models and includes new features, such as state-dependent correlation matrixes and convex risk measures. Crucial to the success of the results are the scenario inputs and especially the mean return assumptions. The model has a number of ways to estimate such scenarios. See also Ziemba [2003] for other scenario estimation procedures and other applications of similar models for various financial institutions. Others have applied stochastic programming models to insurance, such as Mulvey et al. [2000] and Høyland and Wallace [2001], and to individual asset-liability management (ALM) such as Consiglio et al. [2004] and Consigli et al. [2012], and others.

Given good inputs, the policy recommendations can improve current investment practice and provide greater confidence to the asset allocation process. The following quote by Konrad Kontriner (member of the board) and Wolfgang Herold (senior risk strategist) of Innovest emphasizes the practical importance of InnoALM:

> The InnoALM model has been in use by Innovest, an Austrian Siemens subsidiary, since its first draft versions in 2000. Meanwhile it has become the only consistently implemented and fully integrated proprietary tool for assessing pension allocation issues within Siemens AG worldwide. Apart from this, consulting projects for various European corporations and pensions funds outside of Siemens have been performed on the basis of the concepts of InnoALM.
>
> The key elements that make InnoALM superior to other consulting models are the flexibility to adopt individual constraints and target functions in combination with the broad and deep array of results, which allows to investigate individual, path dependent behavior of assets and liabilities as well as scenario-based and Monte-Carlo like risk assessment of both sides.
>
> In light of recent changes in Austrian pension regulation, the latter even gained additional importance, as the rather rigid asset based limits were relaxed for institutions that could prove sufficient risk management expertise for both assets and liabilities of the plan. Thus, the implementation of a scenario-based asset allocation model will lead to more flexible allocation restraints that will allow for more risk tolerance and will ultimately result in better long term investment performance.
>
> Furthermore, some results of the model have been used by the Austrian regulatory authorities to assess the potential risk stemming from less constraint pension plans.

ENDNOTES

Special thanks to Professor Alois Geyer for work with me that led to our joint article on this model, Geyer and Ziemba [2008].

[1]For information, see http://www6.software.ibm.com/sos/features/stoch.htm).

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